MESONIC AND BINDING CONTRIBUTIONS TO THE EMC EFFECT IN A RELATIVISTIC MANY-BODY APPROACH

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Abstract

Starting from a covariant relativistic formalism which uses nucleon and pion propagators in a nuclear medium, we revise the traditional effects of Fermi motion, binding and mesonic corrections to the EMC effect. The calculations are done in a very accurate way, using precise nucleon spectral functions and meson self-energies and a good reproduction of the EMC data is obtained outside the shadowing region which is not explored.

1 Introduction

The EMC effect is one of the most studied and debated processes in the interface of particle and nuclear physics. It thus looks strange that one comes out with new ideas. Actually the ideas used here are not so new, they are those that with the pass of time have survived a thorough scrutiny. Yet, as I will show here, all these ideas were never put together and were not accounted for at the level of precision that the present work has accomplished.

These ideas are the following:

1) Pionic effects, which are relevant around $x = 0.1 - 0.3$ [1].

2) Fermi motion, of relevance at $x > 0.6$ [2].

3) Binding effects, responsible for the dip around $x \simeq 0.5 - 0.6$ [3].

4) Correlation between energy and momentum accounted for by means of nuclear spectral functions [4].

5) Relativistic effects [5].

In connection with these ideas let us see the novelties that our work has contributed [6].

1) We use a relativistic formalism from the beginning. The structure functions are written in terms of nucleon and pion propagators in the nuclear medium and hence we have a covariant relativistic framework. One of the motivations to do so was to avoid the use of the flux factor introduced in Ref. [7] in order to account for relativistic effects in the nonrelativistic calculations. Actually one of our findings is that this prescription is numerically rather inaccurate and should not be used as a substitute of a proper relativistic calculation.

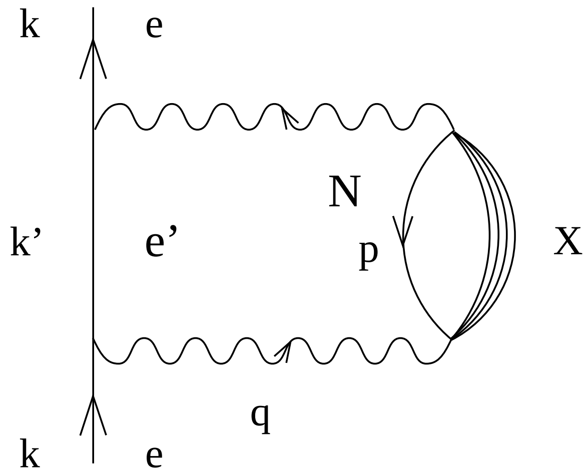

Figure 1: Electron self-energy associated with the process of deep inelastic electron-nucleon scattering.

2) Our approach leads automatically to the formalism of spectral functions used in [4], only that it appears in a relativistic form.

3) Mesonic effects have been reevaluated. Improvements appear in three fronts.

a) Static approximations relating mesonic effects to the “pion excess” are proved to be inaccurate and they are avoided.

b) The results are written in terms of the pion propagator $D(q)$ in the medium. We prove that it is essential that the pion propagator fulfills the sum rule

$$\int_0^\infty \frac{dp^0}{\pi}(-)ImD(p^0,p)2p^0 = 1\,, \tag{1}$$

which is the statement of the equal time commutation of the pion fields expressed in momentum space.

The satisfaction of Eq. (1) requires exact analytical properties of the pion propagator, which most approximations used in the Literature do not fulfill.

c) We included the effects of the ρ-meson cloud for the first time.

2 Formalism

We evaluate the self-energy for the electron corresponding to the diagram of Fig. 1 in infinite nuclear matter.

The probability per unit time of the electron interacting with the nucleons in the medium to give the final state is given by

$$\Gamma(k) = -\frac{2m}{E_e(\vec{k}\,)}Im\Sigma(k) \tag{2}$$

and this can be reconverted into the contribution to the cross section by an element of nuclear volume by means of

$$d\sigma = \Gamma dt dS = \Gamma\frac{dt}{dl}dl dS = \frac{\Gamma}{v}d^3r =$$

$$= \Gamma \frac{E_e(\vec{k}\,)}{k} d^3r = -\frac{2m}{k} Im\Sigma d^3r\,. \tag{3}$$

Hence the cross section for eA scattering is given by

$$\sigma = \int d^3r \left(\frac{-2m}{k}\right) Im\Sigma(\rho(r)) d^3r\,, \tag{4}$$

where we have used the fact that Σ is a function of the nuclear density and we substitute ρ by $\rho(\vec{r})$, the experimental density of the nucleus under consideration. So we are led in a natural way to the local density approximation (LDA) which is a highly accurate tool for this purpose. Indeed, we have also evaluated the structure function using the finite nucleus spectral function of Ref. [8] and found differences of the order of 2 % in the EMC region with respect to the LDA calculation using the same input.

The relativistic nucleon propagator in a medium is given by

$$G(p^0, p) = \frac{1}{\not{p} - M - \Sigma} \tag{5}$$

where

$$\Sigma(p) \equiv \Sigma^s + \Sigma^v \gamma^0 + \Sigma^{v'} \vec{\gamma}\vec{p} \tag{6}$$

We separate the nucleon propagator into the positive energy and negative energy parts as

$$\begin{aligned} G(p^0, p) = \frac{M}{E(\vec{p})} \Bigg\{ \sum_r u_r(\vec{p})\bar{u}_r(\vec{p}) \left[\frac{1 - n(\vec{p})}{p^0 - E(\vec{p}) + i\epsilon} + \frac{n(\vec{p})}{p^0 - E(\vec{p}) - i\epsilon} \right] \\ + \frac{\sum_r v_r(-\vec{p})\bar{v}_r(-\vec{p})}{p^0 + E(\vec{p}) - i\epsilon} \Bigg\}\,. \end{aligned} \tag{7}$$

We omit then the negative energy components and make an expansion of $(\not{p} - M - \Sigma)^{-1}$ in terms of the positive energy part of the propagator of Eq. (7). By doing this, one benefits from the fact that all the terms of the nucleon self-energy of Eq. (6) are diagonal in the spinors u_r. For instance $\bar{u}_r(\vec{p})\gamma^0 u_s(\vec{p}) \propto \delta_{rs}$.

This allows one to write finally an expression for the nucleon propagator in terms of relativistic spectral functions as

$$\begin{aligned} G(p^0, p) = \frac{M}{E(\vec{p})} \sum_r u_r(\vec{p})\bar{u}_r(\vec{p}) \Bigg[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p)}{p^0 - \omega - i\eta} \\ + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, p)}{p^0 - \omega + i\eta} \Bigg] \end{aligned} \tag{8}$$

with a normalization for $S_h(\omega, p)$ given by

$$4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, k_F(\vec{r})) d\omega = A\,, \tag{9}$$

which is obtained by demanding that the baryonic number is A (this was the justification to introduce the flux factor in Ref. [7]).

In omitting the negative energy terms in the propagator we are neglecting zigzag terms contributing to the nucleon self-energy. These, and other possible terms missing in our

approach to the nucleon self-energy are recovered later on by adding a phenomenological piece to the nucleon self-energy such that the binding energy obtained for each nucleus corresponds to the experimental one. Hence our emphasis is in using input as consistent as possible with experimental information in order to make the results highly accurate and as model independent as possible.

3 Deep inelastic cross section

By explicitly evaluating Eq. (4) using the nucleon propagator of Eq. (8) we find

$$\sigma_A = \frac{\alpha^2}{k} \int \frac{d^3k'}{E_e(\vec{k}')} L'_{\mu\nu} W'^{\mu\nu}_A \, , \tag{10}$$

where $L'_{\mu\nu}$ is the leptonic tensor

$$L'_{\mu\nu} = 2k_\mu k'_\nu + 2k'_\mu k_\nu + q^2 g_{\mu\nu} \tag{11}$$

and $W'^{\mu\nu}$ the hadronic tensor given by

$$W'^{\mu\nu} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, p) W'^{\mu\nu}(p, q)$$

$$p \equiv (p^0, \vec{p}); \; W^{\mu\nu} = \frac{1}{2}(W'^{\mu\nu}_p + W'^{\mu\nu}_n) \tag{12}$$

for the nucleonic contribution to the hadronic tensor

Recalling gauge invariance $W'^{\mu\nu}$ can be written in terms of two invariant structure functions.

$$W'^{\mu\nu} = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) W_1 + \left(p^\mu - \frac{p.q}{q^2} q^\mu \right) \left(p^\nu - \frac{p.q}{q^2} q^\nu \right) \frac{W_2}{M^2} \tag{13}$$

In the Bjorken limit ($-q^2 \equiv Q^2 \to \infty, \; q^0 \to \infty$) we define

$$\begin{aligned} x_N &= \tfrac{-q^2}{2p.q} \quad ; \quad \nu_N = \tfrac{p.q}{M} \\ x &= \tfrac{-q^2}{2Mq^0} \quad ; \quad \nu = \tfrac{Mq^0}{M} = q^0 \end{aligned} \tag{14}$$

where the variables x_N, ν_N refer to the nucleons of the nuclear medium and x, ν are the corresponding variables for nucleons at rest in the rest nuclear frame.

One then introduces F_1, F_2, the Bjorken structure functions which in the Bjorken limit depend only on x and are defined as

$$\begin{aligned} \nu W_2(x, Q^2) &\equiv F_2(x) \, , \\ M W_1(x, Q^2) &\equiv F_1(x) \, , \end{aligned} \tag{15}$$

which furthermore satisfy the Callan-Gross relation

$$2x F_1(x) = F_2(x) \, . \tag{16}$$

Eq. (12) in the Bjorken limit becomes

$$F_{2A,N}(x) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,p)\frac{x}{x_N}F_{2N}(x_N)\theta(x_N)\theta(1-x_N)\,, \quad (17)$$

which gives us the nucleonic contribution to the nuclear structure function.

On coming to this point is worth commenting why nuclear effects matter in the EMC effect. One often hears questions from our high energy colleagues of "how can binding and Fermi motion effects matter when one is performing experiments with electrons of hundreds of GeV?". The answer is shown in Eq. (17) since the F_{2N} structure function appears with argument x_N not x, and we can prove that in the Bjorken limit

$$\frac{x}{x_N} = \frac{p^0 - p^3}{M}\,. \quad (18)$$

($\vec{q}$ has been chosen in the z direction). So what Eq. (18) shows is that one is gauging the binding energies and the momenta against the nucleon mass, not against Q^2 or q^0.

For the mesonic contribution one evaluates the electron self-energy corresponding to the diagram of Fig. 2, from where we have to subtract the terms linear in the density since they are already included in the nucleon structure function evaluated in Eq. (17).

Hence, one finds a pionic contribution, from the pion renormalization in the medium, additional to the nucleonic one calculated before, given by

$$F_{2A,\pi}(x_A) = -6\int d^3r \int \frac{d^4p}{(2\pi)^4}\theta(p^0\,)\delta ImD(p)\frac{x}{x_\pi}2MF_{2\pi}(x_\pi)\theta(x_\pi - x)\theta(1-x_\pi) \quad (19)$$

with

$$\frac{x}{x_\pi} = \frac{-p^0 + p^3}{M} \quad (20)$$

and

$$\delta D = D - D_0 - \left.\frac{\partial D}{\partial \rho}\right|_{\rho=0} \rho \quad (21)$$

and a similar expression for the ρ meson contribution.

It is worth discussing briefly which are the approximations implicitly made when one uses the approximation of the "pion excess in nuclei". One can visualize it from our

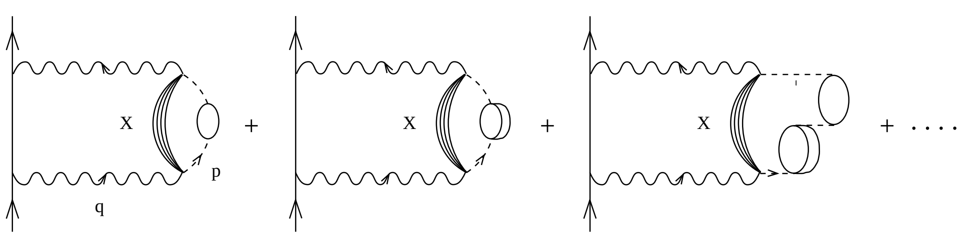

Figure 2: Diagrams of the electron self-energy including $1ph, 1\Delta h, 1ph1\Delta h$, etc..

approach. If we neglect the fact that $F_{2N}(x_\pi)$ depends on p^0 and that one has the strict θ functions from phase space requirements in Eq. (19), we could integrate $Im\delta D$ and have

$$\frac{\delta N_\pi(\vec{p})}{2\omega(\vec{p})} = -3\int_0^\infty \frac{dp^0}{2\pi}\delta ImD(p) \tag{22}$$

and then one would have $F_{2A,\pi}$ as an integral over $\vec{p}$ of the "pion excess" distribution in the nucleus. It is clear that this "approximation" is forcing the contribution of Eq. (19) for values of p^0 forbidden by energy and momentum conservation. Numerically it leads to an overestimate of the mesonic effects and should be avoided.

In Fig. 3 we show now the results for the ratio R_N defined as

$$R_N(x) = \frac{F_{2A,N}(x_A)}{AF_{2N}(x)}\ . \tag{23}$$

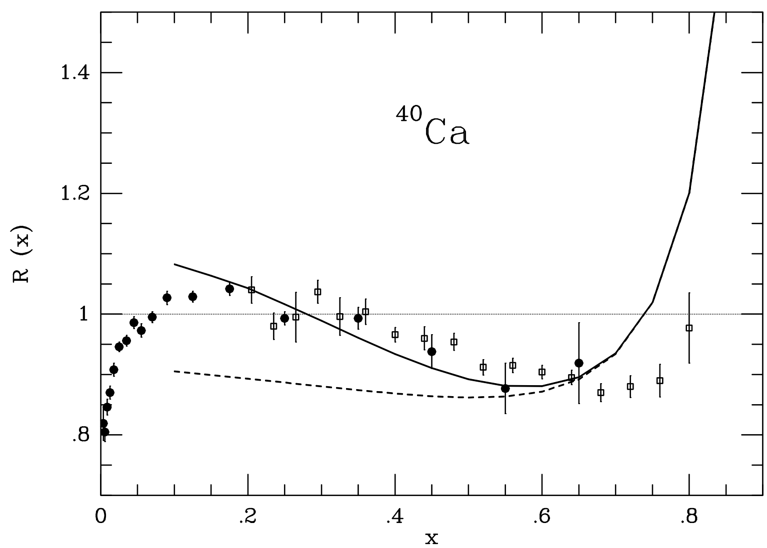

Figure 3: Results for $R(x)$ for ^{40}Ca. Solid lines: whole calculation including the nucleons and the mesons; dashed line: contribution of the nucleons.

We divide by $F_{2N}(x)$, the nucleon structure function, instead of dividing by the deuteron structure function as it corresponds to the data. The reason is that we cannot use the LDA approximation to evaluate the deuteron structure function.

Since the ratio of the deuteron structure function to twice the average one of the nucleon is practically unity up to $x \simeq 0.6$ and then becomes bigger than unity, we should expect an overestimate of the experimental ratio R_N in the $x > 0.6$ region, as it is the case. However, in Refs. [9, 10], where we evaluate the structure function for $x > 1$ we show absolute values which are in agreement with the experiment. The results for ^{40}Ca agree well with experiment outside the shadowing region which we have not studied here. In Fig. 3 the dashed line is the nucleonic contribution alone while the solid line includes also the mesonic effects.

In Fig. 4 we show the same results for ^{56}Fe but we show explicitly the contribution of the pions (intermediate line) and the pions plus ρ-meson, (upper line), (the lower one indicates the nucleonic contribution alone).

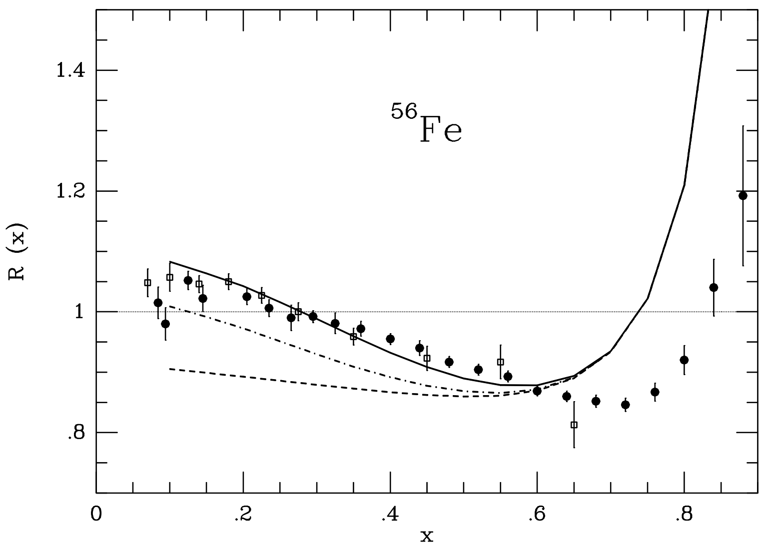

Figure 4: Results for $R(x)$ for ^{56}Fe. Solid line: whole calculation including the nucleons and the mesons; dashed line: contribution of the nucleons; dot-dashed line: contribution of nucleons plus pions.

We observe that the pionic contribution is more moderate than the one found in Refs. [1] or [11]. On the other hand our results for the nucleonic contribution are about 10% lower around $x = 0$ than those of Ref. [12], where a nonrelativistic calculation is done and the flux factor of Ref. [7] is used.

4 Conclusions

We have performed a very accurate calculation of the nuclear structure function in the region of the EMC effect which incorporates the effects which have been found relevant in the past: relativistic effects, binding, Fermi motion and mesonic contributions.

We have put all these elements together for the first time as well as the effects from the renormalization of the ρ-meson cloud.

When all these elements are put together we find a good agreement with experiment outside the shadowing region which is not explored.

This suggests that the nucleon, resonances and mesonic degrees of freedom are an adequate tool to deal with the many-body problem in deep inelastic scattering. Furthermore, we showed that the many-body effects are quite relevant in spite of the large energies of the leptons compared to the scale of the binding nuclear energies and Fermi momenta, and thus must be adequately considered in whichever other framework or other degrees of freedom one chooses to study deep inelastic lepton scattering in nuclei.

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